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Numerical study of the trajectory stability of lateral-abnormal projectiles penetrating soil at small angles of attack

-- Modified Integrated Force Law

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Section 1

Introduction

1. Introduction

accurate prediction of an earth penetrating projectile's trajectory

the Poncelet Analysis Law

the Differential Area Force Law (DAFL)

computational ease

computational accuracy

the Integrated Force Law (IFL)

newly derived equations

increased integral equations accuracy

the Modified Integrated Force Law (MIFL)

1. Introduction

the IFL method		the MIFL method
without the azimuthal angle	⇐ **force and moment integral equations** ⇒	with the azimuthal angle
Adaptive Simpson's method	⇐ **approximate solution** ⇒	Romberg's method
1e-2	⇐ **error tolerance** ⇒	1e-6
ogive nose projectiles	⇐ **projectile geometry** ⇒	various irregular rigid projectiles

Section 2

Geometry of lateral-abnormal projectiles

2.1 Coordinate systems

2.2 Description of Lateral-abnormal projectiles

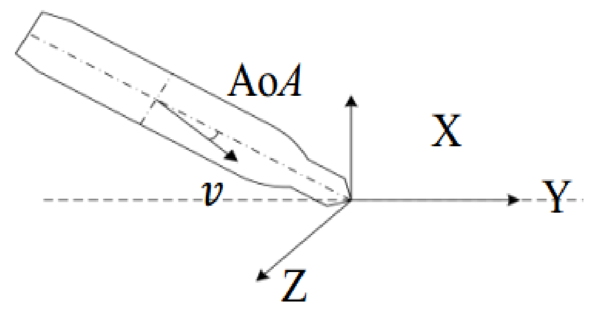
2.3 3D geometry of the later-abnormal projectiles

2.1 Coordinate systems

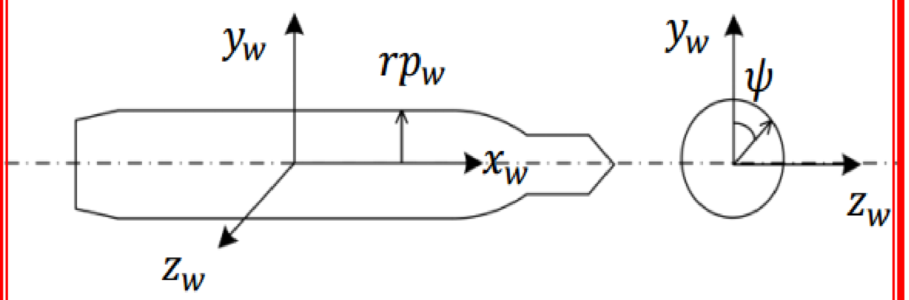
3D problem reduced to 2D problem

the plane motion hypothesis

- **2 coordinate systems**
  - **inertial coordinate system** ⇦ **[X-Y-Z]**
  - **weapon coordinate system** ⇦ **[x_w-y_w-z_w]**
- **3 degrees of free**
  - **translation velocity of CG** ⇦ **[V_x,V_y,0]**
  - **rotation velocity of CG** ⇦ **[0,0,ω_z]**



a) inertial coordinate system



b) weapon coordinate system

2.1 Coordinate systems

Origin: the tip of the projectile impacts the soil
X-axis: soil surface
Y-axis: normal to soil surface

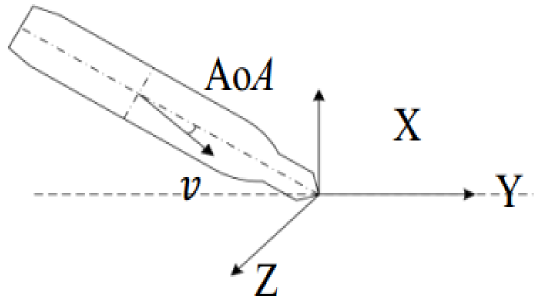
inertial coordinate system

$$T=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

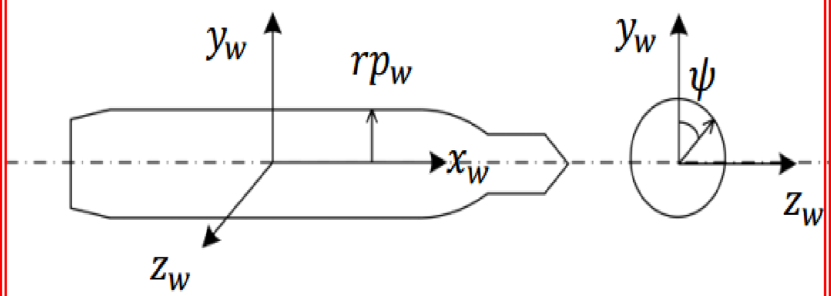
$$T=\begin{bmatrix}\cos(-\theta) & \sin(-\theta)\\ -\sin(-\theta) & \cos(-\theta)\end{bmatrix}$$

weapon coordinate system

Origin: CG of the projectile
x_w-axis: centerline of the projectile
y_w-axis: radius direction of the projectile



a) inertial coordinate system



b) weapon coordinate system

2.2 Description of Lateral-abnormal projectiles

different geometrical nose pin on the front of the penetrator

- conical nose pin ⇦ **projectile #5 and #6**
- blunt nose pin ⇦ **projectile #3 and #4**
- ogive nose pin and others

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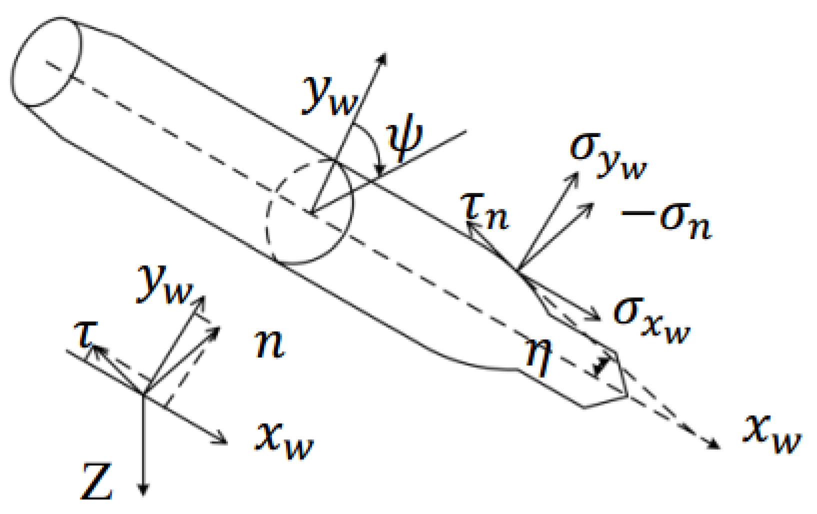
2.3 3D geometry of the later-abnormal projectile



$$f'(x_w) = \tan(\pi - \eta)$$

$$\begin{cases} \sin\eta = \dfrac{-f'(x_w)}{\sqrt{1 + f'(x_w)^2}} \\ \cos\eta = \dfrac{1}{\sqrt{1 + f'(x_w)^2}} \end{cases}$$

$$dS = \sqrt{1 + f'(x_w)^2}\,dx_w$$

$$dA = rp_w d\psi dS = f(x_w)\sqrt{1 + f'(x_w)^2}\,dx_w d\psi$$

Section 3

Modified Integrated Force Law (MIFL) method

3.1 the stress of the contact resistance--SCET

3.2 the force and moment integral

3.3 two-dimensional rigid body dynamics

3.1 the stress of the contact resistance--SCET

projectile penetration velocity vector at point *P*

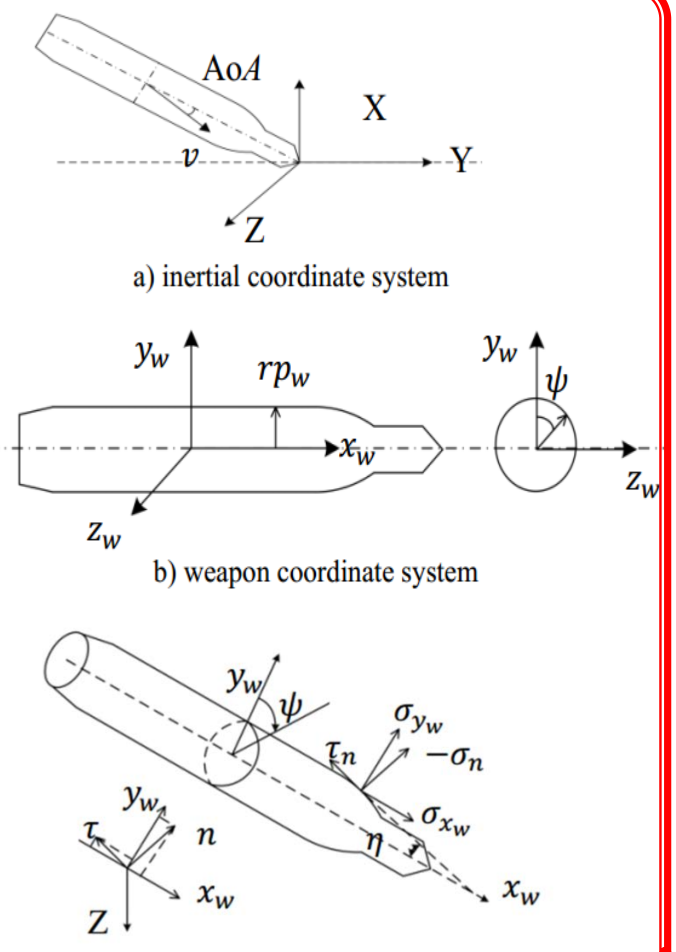
$$\overrightarrow{V_p} = \overrightarrow{V_{cg}} + \overrightarrow{r_P} \times \overrightarrow{\omega}$$

the cavity-expansion velocity at point *P*

$$V_n = \overrightarrow{V_P} \cdot \vec{n}$$
$$\vec{n} = (\sin\eta, \cos\eta)$$

the stress of the contact resistance at point P

$$\sigma_n = \tau_0 A + \rho_0 B V_n{}^2$$
$$\tau_n = \mu \sigma_n \le \tau_0$$



3.1 the stress of the contact resistance--SCET

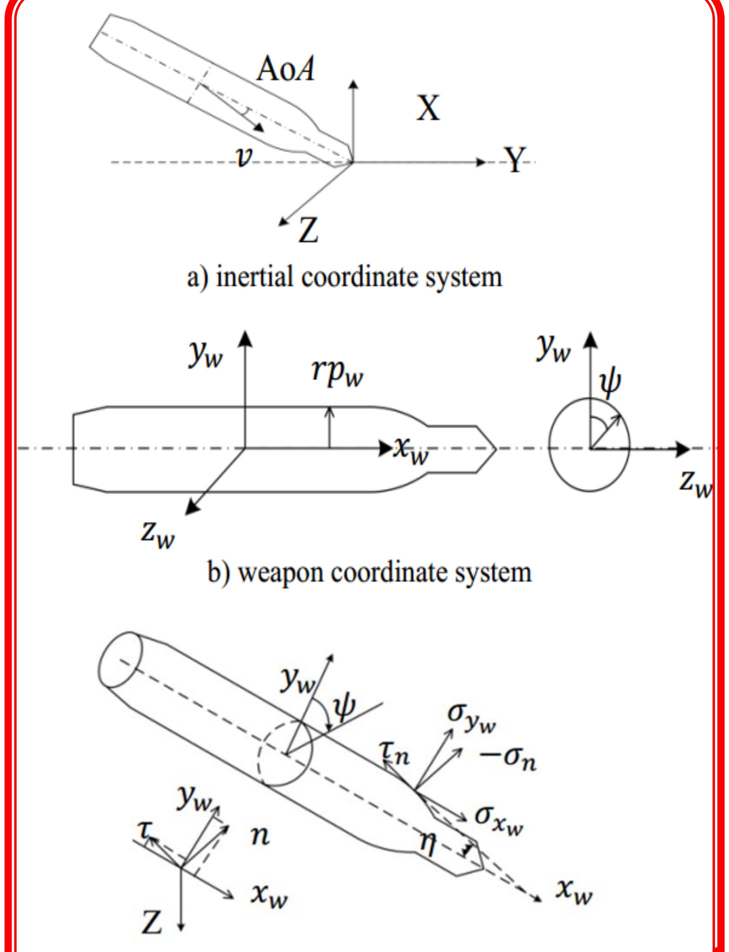
the axial and radial direction

$$\begin{cases}\sigma_{at} = -\sigma_n sin\eta - \tau_n cos\eta \\ \sigma_{rt} = -\sigma_n cos\eta + \tau_n sin\eta\end{cases}$$

the x_w and y_w direction

$$\begin{cases}\sigma_{x_w t} = \sigma_{at} \\ \sigma_{y_w t} = \sigma_{rt} cos\psi\end{cases}$$

$$\begin{cases}\sigma_{x_w t} = \sigma_n(-sin\eta - \mu cos\eta) \\ \sigma_{y_w t} = \sigma_n(-cos\eta + \mu sin\eta)cos\psi\end{cases}$$



a) inertial coordinate system

b) weapon coordinate system

3.2 the force and moment integral

$$\begin{cases} dF_{x_w t} = \sigma_{x_w t} dA \\ dF_{y_w t} = \sigma_{y_w t} dA \end{cases}$$

$$\Downarrow$$

$$\begin{cases} dF_{x_w t} = \sigma_n [f'(x_w) - \mu] f(x_w) dx_w d\psi \\ dF_{y_w t} = \sigma_n [-1 - \mu f'(x_w)] \cos\psi f(x_w) dx_w d\psi \end{cases}$$

the x_w and y_w direction

$\Uparrow$

external force vector component

acting on an infinitesimal top half area

external moment vector component

$\Downarrow$

the z_w direction

$$dM_t = x_w dF_{y_w t} - r p_w \cos\psi dF_{x_w t}$$

$$\Uparrow$$

$$dM_t = x_w dF_{y_w t} - y_w dF_{x_w t}$$

3.2 the force and moment integral

$$\begin{cases} F_{x_wt} = \int dF_{x_wt} \\ F_{y_wt} = \int dF_{y_wt} \end{cases}$$

$\Downarrow$

external force vector component

acting on total top half area

external moment vector component

$\Uparrow$

$$M_{y_wt} = \int x_w dF_{y_wt} - \int y_w dF_{x_wt}$$

$$\begin{cases} F_{x_wt} = (\psi_{t2} - \psi_{t1}) \int \sigma_n [f'(x_w) - \mu] f(x_w) dx_w \\ F_{y_wt} = -(\sin\psi_{t2} - \sin\psi_{t1}) \int \sigma_n [1 + \mu f'(x_w)] f(x_w) dx_w \\ M_t = -(\sin\psi_{t2} - \sin\psi_{t1}) \int \sigma_n [1 + \mu f'(x_w)] x_w f(x_w) dx_w \\ \qquad -(\sin\psi_{t2} - \sin\psi_{t1}) \int \sigma_n [f'(x_w) - \mu] f^2(x_w) dx_w \end{cases}$$

Ignore the separation and reattachment effect

$$\begin{cases} \psi_{t2} - \psi_{t1} = \psi_{b2} - \psi_{b1} = \pi \\ \sin\psi_{t2} - \sin\psi_{t1} = \sin\psi_{b2} - \sin\psi_{b1} = 2 \end{cases}$$

3.3 two-dimensional rigid body dynamics

external force vector

external moment vector

$$\begin{cases} m\vec{a} = \vec{F} \\ J\vec{\beta} = \vec{M} \end{cases}$$

$$\begin{cases} dV_x/dt = F_x/m \\ dV_y/dt = F_y/m \\ d\theta/dt = M/I_z \end{cases}$$

$$\begin{cases} F_x = (F_{x_wt} + F_{x_wb})cos\varphi + (F_{y_wt} + F_{y_wb})sin\varphi \\ F_y = (F_{y_wt} + F_{y_wb})cos\varphi - (F_{x_wt} + F_{x_wb})sin\varphi \end{cases}$$

inertial coordinate system

$$T = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$$

weapon coordinate system

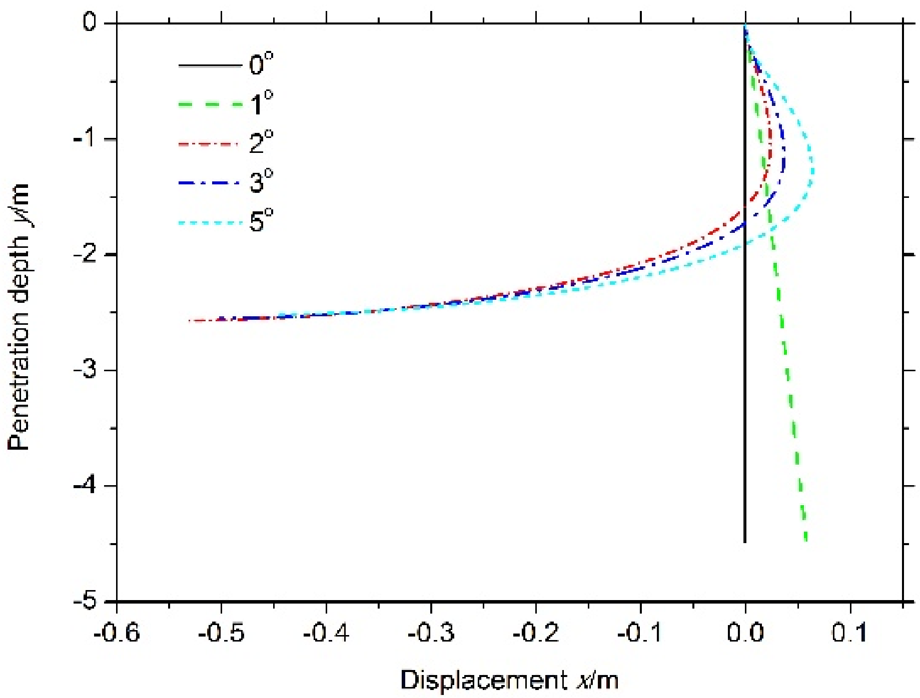
Section 4

Numerical analysis and field test

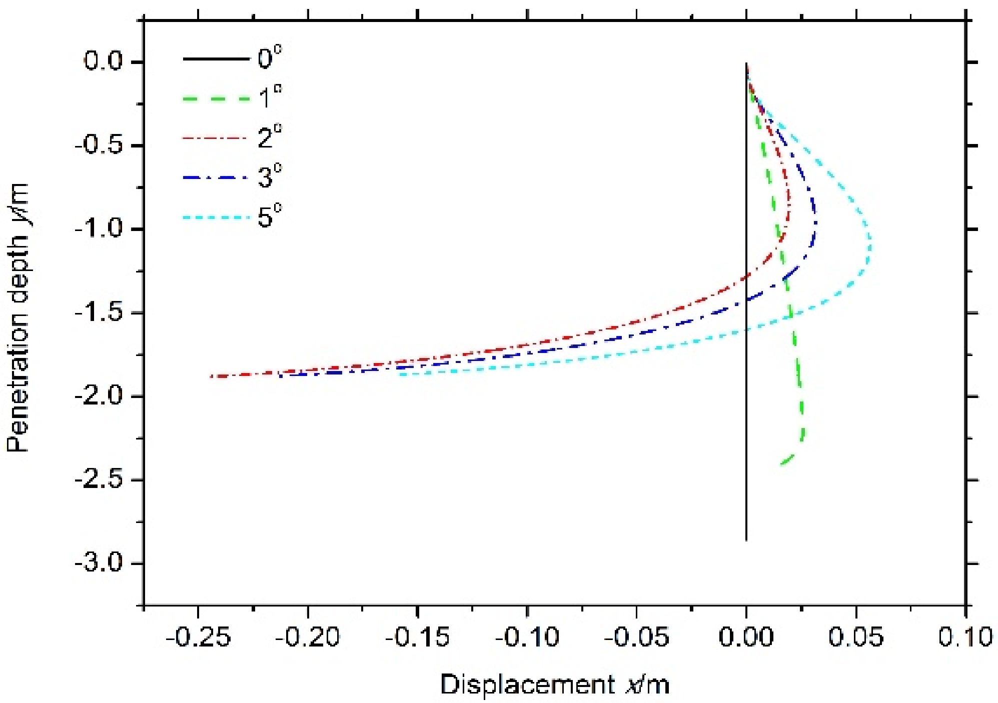
4.1 numerical analysis—trajectory stability

the AOA condition

conical nose pin



blunt nose pin

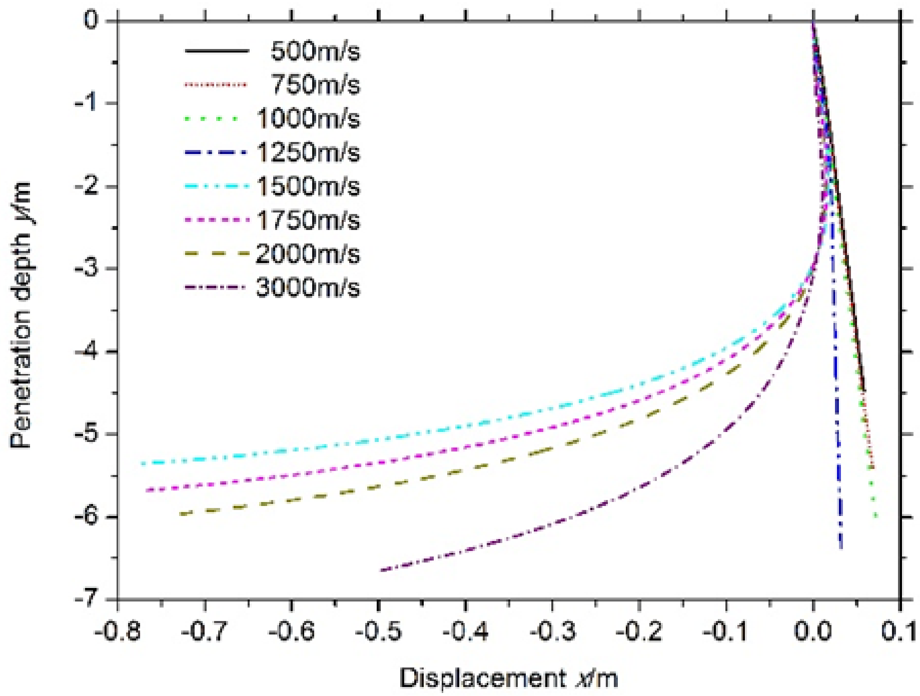


impact velocity $v_i = 500m/s$

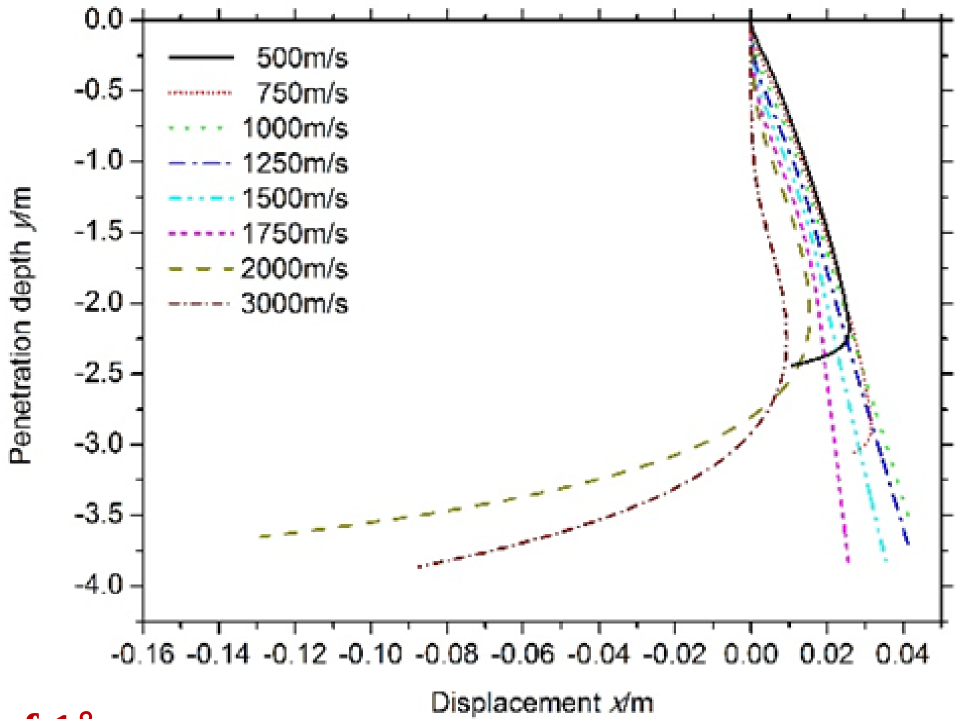
4.1 numerical analysis—trajectory stability

the velocity condition

conical nose pin



blunt nose pin



AoA of 1°

4.2 field test

parameters		condition		results		
		Impact velocity	AoA/°	max penetration depth /m	max later displacement /m	total turning angle /°
test results	conical	486	2~3	2.58	-0.44	160
		508	2~3	2.68	-0.48	154
	blunt	495	2~3	2.15	-0.31	118
numerical results	conical	500	2	2.57	-0.53	98.67
			3	2.56	-0.5	99.33
	blunt	500	2	1.88	-0.25	70.46
			3	1.88	-0.22	70.16

Section 5

Conclusion

Numerical study of the trajectory stability of lateral-abnormal projectiles penetrating soil at small angles of attack

Modified Integrated Force Law

- **the integral equations**
  - various irregular rigid projectiles
  - with the azimuthal angle

⇩

- **Romberg's method**
  - 1e-6

⇩

- **critical reverse conditions**
  - different nose pin geometry
  - the AOA condition
  - the velocity condition

Thank you ！